WIRELESS COMMUNICATIONS AND MOBILE COMPUTING
Wirel. Commun. Mob. Comput. 2006; **6**:1047–1056
Published online 9 January 2006 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/wcm.286

New bit loading algorithms for DMT systems based on the greedy approach

Costas Assimakopoulos*,† and Fotini-Niovi Pavlidou

Department of Electrical and Computer Engineering, Telecommunications Division, Aristotle University of Thessaloniki, Panepistimioupolis of Thessaloniki, 54124 Thessaloniki, Greece

Summary

The performance of a DMT system, which transmits information over channels with varying characteristics through the frequency zones, is improved when the subchannels of the system are loaded with variable data rate. In this study, we meet the bit loading problem of the subcarriers of a DMT system. We propose two new optimum-loading algorithms with low computational complexity. These algorithms assign integer number of bits successively until a target bit rate is fulfilled. We compare them with several existing algorithms in terms of transmitting energy versus data rate and complexity versus system's characteristics i.e. number of subcarriers and target data rate. Simulations prove that without sacrificing optimality both our proposals present a lower complexity level. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: bit loading algorithms; DMT systems; constrained optimization problem

1. Introduction

In wireless multicarrier communications, the transmission of varying number of bits over different subcarriers improves system performance [1]. The adaptation of the subcarrier bit rate to channel characteristics constitutes the basic concept of a loading algorithm. The parameters to be considered in such a loading algorithm are the data rate, the bit error rate and the total transmitting power.

Generally, the loading problem can be formulated as follows: one parameter is being optimized (either maximized or minimized) whereas the other two impose the constraints of the problem. The algorithms proposed in the literature so far aim to maximize the rate and minimize the error rate or the transmission power resulting in a non-integer bit distribution R_i over the i subchannels, but since the quadrature amplitude modulation (QAM) leads to constellations with integer number of bits assigned to every modulation signal, in a last step of the loading algorithm the non-integer number bit distribution has to be rounded to integers [2,3,4].

However, there is a category of loading algorithms that minimize the power or maximize the rate for a fixed error rate assigning to channels an integer number of bits in every step of the algorithm, avoiding the final rounding of the bit distribution [5–9]. These algorithms work based on the so-called greedy approach of the problem i.e. the successive bit assignment to the carriers until R_T bits are assigned. In every algorithmic iteration one bit is added to the carrier that

*Correspondence to: Costas Assimakopoulos, Department of Electrical and Computer Engineering, Telecommunications Division, Aristotle University of Thessaloniki, Panepistimioupolis of Thessaloniki, 54124 Thessaloniki, Greece.
†E-mail: casim@vergina.eng.auth.gr

introduces the minimum overall energy increase aiming to minimize the total transmitting energy.

In this study, two new bit loading algorithms are proposed. Both of them assign bits to the subcarriers of a DMT system iteratively until the constraints are fulfilled. However, the procedure of the bit assignment is accelerated since the bits are not distributed one by one but in groups. This is the major innovation that the described algorithms introduce. The first proposal works on the table of the incremental energies $\Delta P_{i,j}$ (see Figure 1). The second proposal, which is even more expeditious than the first concerning the number of comparisons, works on the normalized incremental energies $\Delta P_{i,j_{\text{normalized}}}$. These quantities are proved in Section 4 that they enclose information about the number of bits j_1 that subcarrier i can be loaded with when the best subcarrier of all in terms of CNR (channel to noise ratio) is loaded with known bit load j_2.

In the following sections, the greedy approach is analyzed and the conditions under which it reaches the optimal solution are explained. Then, the two new bit-loading algorithms are described step by step, examining the complexity that is inserted. Finally, a comparison of several existing algorithms and our proposals are given on the same transmission channel. The comparisons are made in terms of transmitting energy versus data rate and complexity versus system's characteristics i.e. number of subcarriers and target data rate. Diagrams expressing complexity versus system parameters show the improvement that our proposals introduce compared to the well-known algorithms.

It should be clarified that in the rest of the paper, perfect channel estimation is assumed and channel variations are slow and followed accurately by the channel equalizer.

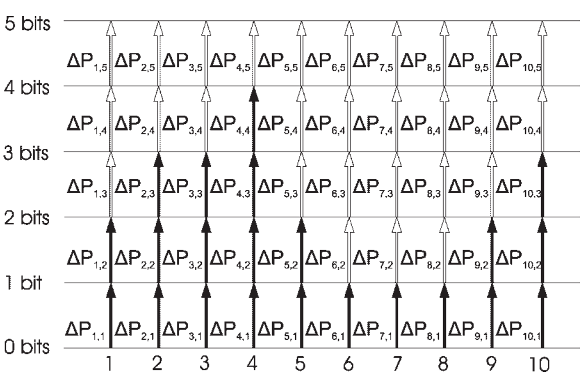

Fig. 1. An example of a DMT system with ten subcarriers and maximum allowable number of bits/subcarrier equal to 5.

2. The Greedy Approach

Assume that we aim to load every DMT symbol with R_T bits, for a predetermined BER transmitting as less energy as possible. Let us denote the jth bit conveyed by the ith subcarrier by the ordered pair (i,j) [9]. This ordered pair has a respective weight $\Delta P_{i,j}$ corresponding to the incremental energy that is added to the system's aggregate energy consumption. Hence, the constrained problem is formulated as follows: Minimize $P_T = \sum \Delta P_{i,j}$ when the summation of j of the pairs(i,j) for $i = 1, \ldots, N$ $j \in [1, R_{\text{m}}]$ is greater than R_T. N is the number of subcarriers and R_{m} the maximum allowable number of bits per carrier. This is clearly an optimization problem. When searching for the optimal solution of such a problem search may progress in one of several ways, depending on the structure of the problem. There are three major approaches: exhaustive search, dynamic programming and greedy. We will examine the latter. Its principle is simple. One starts with a partial non-feasible solution and at each stage a partial solution is maintained iteratively until a feasible solution has been found [10,11]. The partial solution is searched in sets of partial solutions one of which is known to lead to the optimal solution. The greedy procedure produces the optimal solution under certain characterization of the sets of the partial solutions. Those sets must be parts of a matroid [10 pp. 390, 12]. A matroid is a family of sets that have some special properties. One of the equivalent definitions of the matroid follows. Let $E = \{e_1, \ldots, e_n\}$ be a finite set of n elements. We call a matroid on E a pair $M = [E, \Phi]$ where Φ is a family of subsets of E satisfying the following axioms:

(1) $\emptyset \in \Phi$ where $\emptyset$ is the empty set.
(2) If $X \in \Phi$ and $Y \subseteq X$ then $Y \in \Phi$.
(3) If U, V are members of Φ with $|U| = |V| + 1$ there exists $e_i \in U \backslash V$ such that $V \cup \{e_i\} \in \Phi$.

If we accomplish to prove that our problem has a matroid structure, then a greedy algorithm reaches a solution that is guaranteed to be optimal. Recall the pairs (i,j) defined in the beginning of this section and collect all these pairs into the set $E = \{(i,j) : 1 \leq i \leq N, \quad 0 \leq j \leq R_{\text{m}}\}$. Let us construct the family Φ of sets of E with cardinality not greater than R_T i.e. $\Phi = \{F \subseteq E : |F| \leq R_T\}$. The pair $M = [E, \Phi]$ is proved in Reference [9] to be a matroid and hence a greedy algorithm working on that matroid will reach the optimum solution.

The simplest greedy algorithm is to distribute the bits one at a time at the subcarrier that introduces the least additional energy to the system. Thus, in every iteration, the algorithm should perform successive comparisons until the best subcarrier is located. The algorithm will execute R_T iterations until all bits are assigned. An algorithm based on this fundamental structure seems to have a complexity that is bounded by R_T iterations, as this is a minimum number of iterations needed to assign R_T bits.

In Reference [10 pp. 393], a greedy algorithm is proposed to work on the elements of the set E of the matroid M that are in order of increasing weights. The algorithm (see Appendix A) reaches the optimum solution, i.e. a set F with $|F| = R_T$, which has the minimum weight $P_T = \min W\{F\}$ compared to all the other sets $F' \in \Phi$ with $|F'| = R_T$.

There is also one last thing that has to be stated before proceeding to Section 3. Let us assume a DMT system with ten subcarriers and maximum QAM constellation size that can support up to 5 bits/subcarrier. Figure 1 shows the incremental energies that are required for a transition of each subcarrier from a lower QAM mode to the next one. The bold arrows are used to distinguish those $\Delta P_{i,j}$ that consist of the optimum final bit distribution. We should emphasize here that for every subcarrier k the distribution cannot be non-continuous. In other words, in the increasing order of the weights it should not be $\Delta P_{k,j+1} < \Delta P_{k,j}$. This is ensured by Proposition 1.

Proposition 1. If $\Delta P_{k,j+1}$ belongs to the optimum distribution then $\Delta P_{k,j}$ belongs to it as well. For the proof of this proposition refer to Appendix B.

Proposition 2 that follows originates directly from the matroid structure of the optimization problem and Proposition 1.

Proposition 2. Suppose that we aim to transmit R_T bits minimizing the total transmitting energy. The optimum bit distribution will consist of the R_T smallest $\Delta P_{k,z}$ out of all $\Delta P_{i,j}$ $i = [1, \ldots, N]$, $j = [1, \ldots, R_{\mathrm{m}}]$ otherwise the distribution would not be optimum.

Based on these two propositions, we describe a simple greedy bit loading algorithm.

3. Proposal 1

3.1. Description of the Algorithm

Note that the criterion for a subchannel selection in order to add one more bit is based on the incremental energies $\Delta P_{i,j}$. Therefore, the calculation of the $\Delta P_{i,j}$ seems to be unavoidable. For a system with N subcarriers that can carry up to R_{m} bits per carrier, we have NR_{m} calculations according to Equation (1):

$$\Delta P_{i,j} = \frac{\mathrm{SNR}(P_{\mathrm{e}}, j) - \mathrm{SNR}(P_{\mathrm{e}}, j-1)}{g_i} \qquad (1)$$

where g_i is the signal to noise ratio of the ith subchannel for unit transmitting energy and $\mathrm{SNR}(P_{\mathrm{e}}, j)$ is the signal to noise ratio that is necessary to guarantee a probability of error less or equal to P_{e} for a QAM system allocating j bits. Thus, we formulate vector $\mathbf{y}$.

$\mathbf{y} = [\Delta P_{1,1}^{(1)}, \Delta P_{2,1}^{(2)}, \ldots, \Delta P_{N,1}^{(N)}, \Delta P_{1,2}^{(N+1)}, \Delta P_{2,2}^{(N+2)}, \ldots, \Delta P_{N,2}^{(2N)}, \ldots, \Delta P_{N,R_{\mathrm{m}}}^{(N \cdot R_{\mathrm{m}})}]$ consisting of the successive N-ary sets of the incremental energies. Every set respects the transitions of all subchannels from a QAM mode to the next one. So, we will have R_{m} sets. The ranking position k of the $\Delta P_{i,j}^{(k)}$ element in vector $\mathbf{y}$ can provide the subcarrier to whom $\Delta P_{i,j}^{(k)}$ belongs to using Equation (2)

$$i = \mathrm{mod}(k, N) \qquad (2)$$

Index k can also give the jth QAM mode that $\Delta P_{i,j}^{(k)}$ refers to using Equation (3)

$$j = \lceil k/N \rceil \qquad (3)$$

where $\lceil x \rceil$ is the smaller integer that is greater or equal to x.

According to Proposition 2 of Section 2, we need the R_T smallest $\Delta P_{i,j}$. Thus, we have to sort the elements of vector $\mathbf{y}$ in ascending order. This way we formulate vector $\mathbf{w}$. Simultaneously, we keep in vector $\mathbf{p}$ the initial ranking positions that the sorted elements of vector $\mathbf{w}$ used to have in $\mathbf{y}$. That is, $w(k) = y(p(k))$. Obviously vector $\mathbf{w}$ does have in its first R_T positions, the incremental energies $\Delta P_{i,j}$ that belong to the optimum distribution. Thus, we managed to locate the $\Delta P_{i,j}$ of interest without any algorithmic iteration apart from the sorting.

Beginning from the greatest incremental energy $k = R_T$ and moving towards the smallest $k = 1$, we assign $j = \lceil p(k)/N \rceil$ bits to the $i = \mathrm{mod}(p(k), N)$ subcarrier. Obviously, if $\mathrm{mod}(p(k), N) = 0$ then $i = N$. Furthermore, based on Proposition 1 of Section 2, if the ith subcarrier has already been assigned bits, then it does not need to be assigned with bits any more.

Alternatively, in order to avoid the divisions that are introduced by Equations (2) and (3), we could add a

Table I. Complexity analysis of the examined algorithms in the literature and our proposals.

	Number of comparisons	Number of divisions/multiplications/ additions/subtractions	Table accesses	Extra memory
H–H	$(N-1)R_T$	R_T+N-1	R_T+N-1	—
Lai *et al.*	$N\log_2(N)+R_T(R_\mathrm{m}-1)$	$R_T+R_\mathrm{m}-1$	$R_T+R_\mathrm{m}-1$	N
Proposal 1	$R_T\log_2(NR_\mathrm{m})$	NR_m	$NR_\mathrm{m}+2N$	$2N$
Proposal 2	$2N\log_2N+N$	$N+N$	—	N
Czylwik	$(R_T-m_s)N+2N\times I$	$N\times M^{\S}$	N	—
Piazzo	$N\log_2N+(R_\mathrm{m}-2)\log_2R_T$	$8(R_\mathrm{m}-2)\log_2R_T$	—	N
Krongold *et al.*	$NR_\mathrm{m}\log_2(N\times R_\mathrm{m})$	$(4N+1)\log_2(N\times R_\mathrm{m})$	NR_m	—
Fischer–Huber	$I_1+N\log_2N+I_2\times D$	$N\times M$	N	—
Sonalkar	$(N-1)L$ where L is equal to $(R_\mathrm{m}N-R_T)$ or imposed by the PSD mask and R_T	$R_\mathrm{m}N-R_T+N-1$	R_T+N-1	—
Baccarelli	$(N-1)R_T$	N	—	—
Campello	$(N-1)R_T$	R_T+N-1	R_T+N-1	—

§The algorithm comprises the calculation of logarithms. M is the order of the complexity for calculating one logarithm.

buffer that would store for every $\Delta P_{i,j}$, the subcarrier i and the modulation mode j that $\Delta P_{i,j}$ refers to, in two successive memory cells. Then vector **p** would consist of pointers locating the address of the first byte of the buffer with respect to every $\Delta P_{i,j}$.[‡] One memory cell would be used to store the subchannel's identity number and a second memory cell to store for that specific subchannel the optimum QAM mode. Thus, the extra memory that is needed is $2\times N$, which is relatively small compared to the avoidance of R_T+N divisions.

3.2. Complexity Analysis

The initial calculation of the $\Delta P_{i,j}$ includes NR_m divisions provided that the $\mathrm{SNR}(P_\mathrm{e},j)$ were precomputed and stored in a table matrix. Equal table accesses are also necessary in order to supply Equation (1) with the appropriate SNR. At this point, we should mention that a table access is a rather quick procedure for contemporary processors and that is why in some papers table accesses are not counted in the complexity evaluation.

The sorting part of the algorithm consists of $NR_\mathrm{m}\log_2NR_\mathrm{m}$ comparisons. The proposed algorithm can be further accelerated if the sorting algorithm stops just after it locates the R_T smallest $\Delta P_{i,j}$ avoiding to sort $NR_\mathrm{m}-R_T$ elements unnecessarily. This could result in a complexity on average equal to $R_T\log_2(NR_\mathrm{m})$.

Finally, the bit allocation leads to R_T divisions at maximum in order to find all the subcarriers, whereas the allocation of bits includes N divisions. Alternatively, using the buffer with size $2\times N$, we can avoid the last R_T+N divisions, increasing the table accesses by a factor equal to the memory increase. The algorithm's complexity is summarized in Table I.

‡The memory increase for a complexity reduction is a widespread method used in numerical analysis. This method is used for example by the sorting algorithms in order to decrease complexity from $O(N^2)$ to $O(N\log_2N)$.

4. Proposal 2

4.1. Description of the Algorithm

This second algorithm is in fact the first optimum proposal whose development is accelerated. From the description of the first proposal, we see that the major complexity is introduced by the sorting of the $\Delta P_{i,j}$ elements. If we knew the sorted position of each $\Delta P_{i,j}$ relatively to the others and all of them relatively to a reference position then this information could accelerate the location of the smallest $\Delta P_{i,j}$.

From Equation (1) for two subchannels k,l that carry j bits and satisfy $g_k>g_l$ we get $\Delta P_{k,j}<\Delta P_{l,j}$. Thus, the sorted g_i can give us information about the relative positions of the incremental energies when the subcarriers are on a common QAM mode (i.e. the same j in Equation (1)). Also, for the same subcarrier k, we have proved in Appendix B that

$$\frac{\Delta P_{k,j+1}}{\Delta P_{k,j}}=2 \tag{4}$$

The ratio of relation (4) (which is independent of g_i) seems to determine when the kth carrier will change mode from j to $j+1$. A thorough explanation follows.

Let us assume that we sort the g_i in descending order $g_1>g_2>g_3>\cdots>g_N$. We start to assign bits successively. The first bit will be allocated to the 1st subcarrier, which will transit from 0 to 1 QAM mode.

The second bit will be allocated to the second subcarrier for which we will have also the transition from 0 to 1 QAM mode. The first subcarrier will take its second bit before the kth subcarrier will get its first bit if

$$\Delta P_{1,2} < \Delta P_{k,1} \rightarrow \frac{\mathrm{SNR}(P_\mathrm{e},2) - \mathrm{SNR}(P_\mathrm{e},1)}{g_1} < \frac{\mathrm{SNR}(P_\mathrm{e},1) - \mathrm{SNR}(P_\mathrm{e},0)}{g_k} \rightarrow \frac{g_1}{g_k} > \frac{\mathrm{SNR}(P_\mathrm{e},2) - \mathrm{SNR}(P_\mathrm{e},1)}{\mathrm{SNR}(P_\mathrm{e},1) - \mathrm{SNR}(P_\mathrm{e},0)} \quad (5)$$

Using the upper bound of probability of error for j mode QAM [13] (whose validity limits are shown in Appendix B) we get:

$$P_\mathrm{e} = 4Q\left(\sqrt{\frac{3\mathrm{SNR}}{2^j - 1}}\right) \rightarrow \mathrm{SNR}(P_\mathrm{e}, j) = \left[Q^{-1}\left(\frac{P_\mathrm{e}}{4}\right)\right]^2 \frac{2^j - 1}{3} \quad (6)$$

From Equations (5) and (6), we get after some manipulations $(g_1/g_k) > 2^{2-1}$.

Thinking in the same way, the 1st subcarrier will get its jth bit before the mth get its first if

$$\frac{g_1}{g_m} > 2^{j-1} \quad (7)$$

We notice that if we get the ratios g_1/g_i, then we have information about the positions of the $\Delta P_{i,j}$ relatively to the best subchannel and additionally if $\Delta P_{m,1}$ belongs to the final optimum distribution and inequality (7) is satisfied, then $\Delta P_{1,j}$ also belongs to it.

Hence, if we compare the sorted order of $\frac{g_1}{g_i}$ with the constants 2^{j-1} for $j = 2, 3, \ldots$ then we get information about the QAM mode switching of the 1st subcarrier.

In the general case, with $k < l$ the kth carrier will get j_2 bits before the lth carrier gets j_1 bits if

$$\Delta P_{k,j_2} < \Delta P_{l,j_1} \rightarrow \frac{g_1/g_l}{g_1/g_k} > 2^{j_2 - j_1} \quad (8)$$

Please notice that number 2 raised in the power $j_2 - j_1$ can be used to determine the QAM mode switching for every carrier.

Before describing the steps of the second proposal, it will be useful to recall from Appendix B that

$$\Delta P_{i,j} = \frac{\{Q^{-1}(P_\mathrm{e}/4)\}^2}{3} \frac{2^{j-1}}{g_i} \quad (9)$$

If we multiply all $\Delta P_{i,j}$ with $3g_1/[Q^{-1}(P_\mathrm{e}/4)]^2$ then we take the normalized incremental energies

$$\Delta P_{i,j\,\mathrm{normalized}} = \frac{g_1}{g_i} 2^{j-1} \quad (10)$$

without affecting their relative position. Please notice again that the normalized incremental energy referring to the QAM mode transition $j-1 \rightarrow j$ equals the

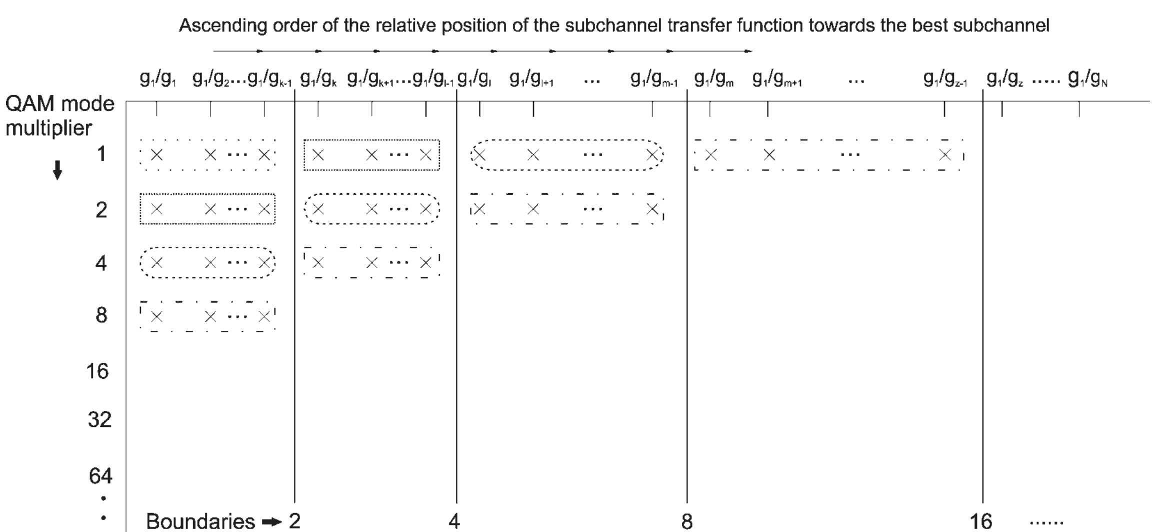

Fig. 2. Successive bit allocation to the subcarriers according to their relative position towards the best subchannel. Within the same frame are the groups of the bits that are allocated in every algorithmic cycle. When the algorithm ends, the $R_\mathrm{tot} - R_T$ greatest $\Delta P_{i,j}$ will be searched among the last allocating group.

multiplication of the relative position of g_i towards g_1 with 2 raised to the power $j-1$.

Based on these arguments, we propose a short algorithm that leads to the optimum bit distribution.

Suppose that we have sorted g_i in descending order. Then we divide g_1 successively with g_i forming the ratios g_1/g_i of Figure 2. Compare the ratios g_1/g_i with the constant boundary $= 2^1$. When $g_1/g_k >$ boundary then the subcarriers $i = 1, \ldots, k-1$ are loaded with their first bit. If $R_T > k-1$, then the bit distribution should continue. We update the boundary $= 2^2$ and continue the comparisons. Now the carriers that have their relative positions towards g_1 is greater than 2^1 and are loaded with their first bit. Simultaneously, the carriers for $i = 1, \ldots, k-1$ are candidates to be loaded with a second bit because this transition will be equal to the multiplication of their relative position towards g_1 with 2^1 (see Equation (10)). Thus, the algorithm goes on comparing the $g_1/g_k, g_1/g_{k+1}, \ldots$ with the new boundary $= 2^2$ until we find the lth subcarrier for which we have $(g_1/g_l) > 2^2$. At that point, the algorithm has distributed $R_{\text{tot}} = k-1+l-1$ bits. If $R_T > R_{\text{tot}}$ then we update boundary $= 2^3$ and continue the comparisons. The bit distribution is made in groups as Figure 2, shows. In Figure 2, every group is rounded by a different frame. If $R_T < R_{\text{tot}}$ then we allocate $R_{\text{tot}} - R_T$ redundant bits that have to be rejected. These bits belong to the $R_{\text{tot}} - R_T$ greatest $\Delta P_{i,j}$ located in the last group assignment (Figure 2 with the dash-dotted frame). All the other $\Delta P_{i,j}$ of the distribution have a relative position towards $\Delta P_{1,1}$ that is smaller than the penultimate boundary, whereas the $\Delta P_{i,j}$ of the last group are greater compared to the penultimate boundary. In Figure 2, the algorithm stops when boundary $= 16$ and the penultimate boundary $= 8$.

Obviously, if we calculate the $\Delta P_{i,j_{\text{normalized}}}$ of the last group and then sort them in ascending order, then we can easily distinguish the $R_{\text{tot}} - R_T$ greater and subtract one bit from the respective subcarriers. This is the second sorting that the evolution of the algorithm imposes. A flow chart of the algorithm is shown in Figure 3.

At this point, we should mention that if a maximum QAM constellation size imposes an upper bound to the number of bits that a subcarrier can be loaded with, then carriers that obtain this maximum number of bits are excluded from the continuation of the algorithm. Because of the nature of the development of the algorithm, we expect these carriers to be located at the lower part of the array of g_1/g_i.

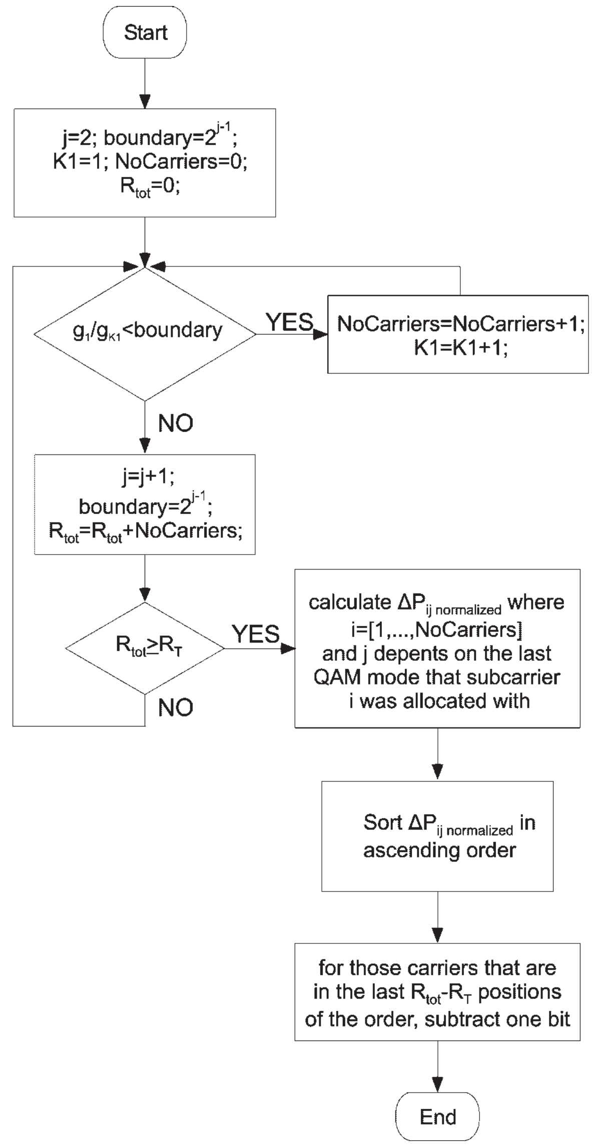

Fig. 3. Flowchart of Proposal 2.

4.2. Complexity Analysis

In the first part, the algorithm begins with the formulation of the order $g_1/g_1, g_1/g_2, g_1/g_3, \ldots, g_1/g_N$. So, this part entails $N\log_2 N$ comparisons and N divisions. In the second part, we have successive comparisons of the g_1/g_i with the updated boundary. In the case that all subcarriers are used we have at maximum N comparisons.

Finally, in the last part the algorithm ends formulating $\Delta P_{i,j_{\text{normalized}}}$ and sorting them in ascending order. If z is the number of the subcarriers of the last group, then this part introduces z multiplications and $z\log_2 z$ comparisons. In the worst case, it should be $z = N$ so the worst scenario introduces N multiplications and $N\log_2 N$ comparisons. Table I summarizes the complexity of the three parts of the algorithm.

5. Performance Comparison of Existing Algorithms and our Proposals

5.1. Energy Versus Data Load

We have simulated some important algorithms known in the References [3,5–9,14,16,17] and our proposals. The algorithms simulated worked on a target data rate mode and we have plotted all of them on Figure 4. The performance comparison is made in terms of the normalized energy transmission per bit for different data rates per DMT symbol. The algorithms present almost the same performance with the exception of Piazzo's algorithm which is suboptimal (it presents a deteriorated performance compared to the others that fluctuate between 0.2 and 0.3 dB on average) as expected [14]. In fact, zooming in the figure the systems present slight differences but they are not worth mentioning, or in other words, this performance comparison is not critical. The fact that optimal and near-optimal bit loading algorithms do not present important performance difference is also shown in Reference [4]. Hence, algorithms performance should be judged from the amount of complexity they introduce.

5.2. Complexity Comparison

In Table I we summarize the complexity analysis of our proposals and some of the existing algorithms [3,5–9,14,16,17]. The following diagrams present the complexity comparison of our systems and the existing algorithms for different data loads and constant R_m, N.

Figure 5 presents the number of comparisons for a system with $N = 256$ subcarriers, maximum QAM mode $R_m = 8$ bits per subcarrier and different data loads per DMT symbol. Hughes–Hartogs (H–H) algorithm is orders of magnitude more complex. Proposal 1 and the algorithm given in Reference [6] have an intersection point. That point can be easily calculated using Table I. In fact the intersection point expresses the data load where both algorithms have the same number of comparisons. That critical load is:

$$R_{T_C} = \frac{N\log_2 N}{\log_2(NR_m) - R_m + 1} \tag{11}$$

The proposed algorithm 1 presents better performance for loads smaller than R_{T_C} whereas for loads greater than R_{T_C} Lai algorithm prevails. The data rate fluctuated from 100 bits per DMT symbol up to 1900. For our system, that could have a maximum load $NR_m = 2048$ bits per symbol, this means that it was shoved to its limit.

For Czylwik's algorithm, we found that the real number bit distribution was only $(R_T - m_s) = 50$ bits away from the final optimum bit distribution on average whereas the power optimization resulted in $I = 20$ row searches among the subcarriers. For Fischer's *et al.* (F–H) algorithm, we found that $I_1 = 10$ carriers had to be turned off on average and $I_2 = 20$ bits was missing from the real number bit

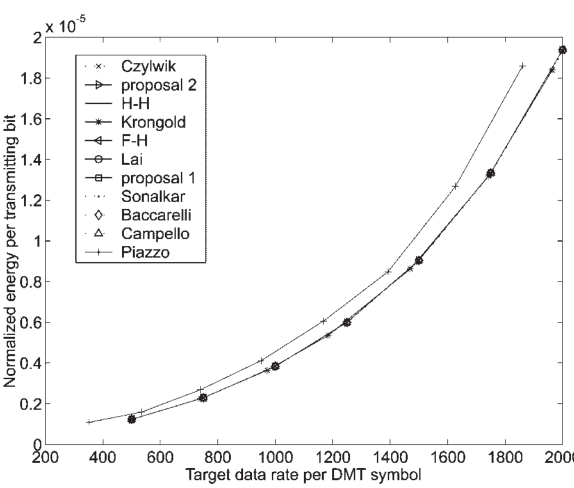

Fig. 4. Normalized transmitting energy per bit for different target data rates.

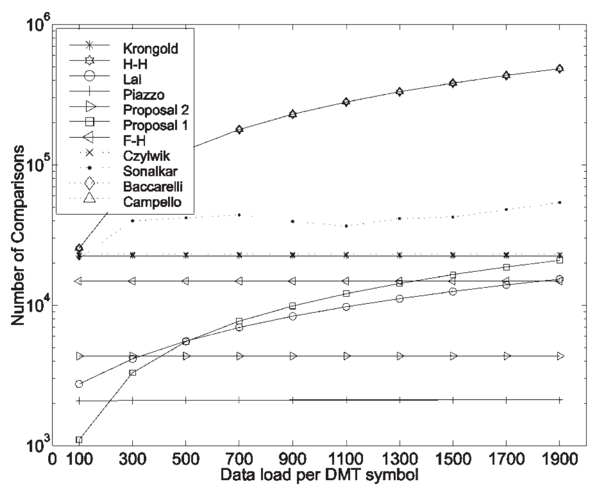

Fig. 5. Number of comparisons for different data loads per DMT symbol.

distribution until the target bit rate was fulfilled. These values were found after running the simulations over several frequency dependent channels.

From Figures 4 and 5, we can easily observe a trade-off between optimization of the transmitting energy and complexity. Piazzo sacrifices transmitting energy for complexity reduction. All the others are close to the optimum distribution and present higher complexity. Proposal 2 is closer to Piazzo than anybody else. So, if Piazzo introduced a low bound of complexity, then Proposal 2 gets close to this boundary without sacrificing optimality.

Except Krongold, F–H and proposal 2, all other algorithms depend on data load fluctuations.

Figure 6 presents the number of divisions, multiplications, additions and subtractions versus target data rate. Proposal 2 still remains the closest to Piazzo's complexity whereas simultaneously H–H algorithm is not any more orders of magnitude away from the others. Concerning Figure 6, we observe that Baccarelli *et al*. present less number of calculations than Piazzo. This is due to the iterative relation that Reference [9] proposes for the calculation of the incremental energies. However, as Reference [9] also suggests this is only the case of an AWGN channel. We should mention here that the number of comparisons is of greater importance and in fact it is the critical quantity for determining complexity as it consumes much more CPU's time. That is why in many papers the complexity analysis lies only on the first column of Table I. Arithmetic calculations that were common to all of the examined algorithms like counter additions were excluded from column 3 of Table I.

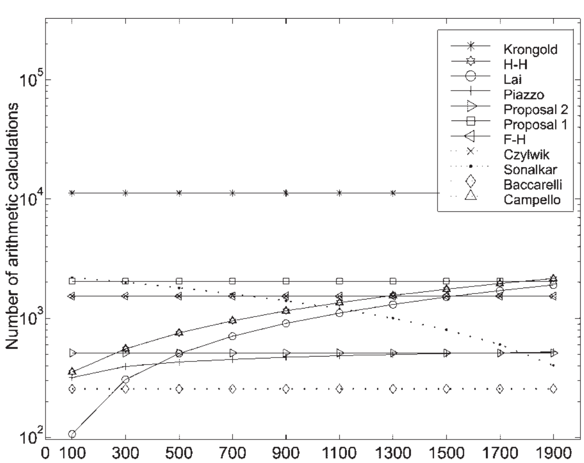

Fig. 6. Number of arithmetic calculations of several algorithms versus target data rate.

6. Conclusions

In this paper, two new bit loading algorithms based on the greedy approach are proposed. Both of them reach the optimum integer bit distribution. The proposals substantially accelerate the bit assignment by distributing groups of bits to the sorted subcarriers (according to the subchannels' CNR). Basically, the well-known bit loading algorithms are divided into two categories: the optimal and the sub-optimal. Optimality refers to the systems performance in terms of transmitting power and achieved data rate. However, it is proved that the sub-optimal algorithms performance is close to the optimal in a way that optimality is not a key feature to choose an appropriate algorithm. Thus, computational complexity plays the main role. A thorough complexity analysis in Section 5 gives an overview of the computational load of many of the well known algorithms existing in the literature. Moreover, it reveals the significant improvement that our proposals introduce and quantify that improvement in Figures 5 and 6.

Appendix A: The Greedy Algorithm [10 pp 393]

(a) To begin with $F^0 = \emptyset$. Examine the elements $e_1, \ldots, e_n$ of the matroid successively in order of increasing weights.
(b) At stage k the kth element e_k is being examined.

Let F^{k-1} be the independent subset obtained at stage $k-1$.

If $F^{k-1} + e_k \in \Phi$, set $F^k = F^{k-1} + e_k$. Else if $F^{k-1} + e_k \notin \Phi$, put $F^k = F^{k-1}$.

If $k < R_T$ set $k = k + 1$ and return to (b). Else if $k = R_T$ stop: $F = F^{R_T}$ is the basis of minimum weight.

We see that the greedy algorithm contains a test of independence at each stage. Hence, it implies that we have a method to check whether any subset of E does or does not belong to the family Φ of independent subsets. In the case of the matroid defined in Section 2, since all the sets having rank less or equal to R_T are included in the family Φ, then it is ensured that $F^{k-1} + e_k \in \Phi$ when $k < R_T$.

Appendix B: Validity Limits of Relation (6)—Proof of Proposition 1

Assume that QAM modulation is chosen to load j bits over a carrier i that presents CNR g_i. The noise disturbance is AWGN. It is shown in Reference [15] that the probability of error in this case is given by

$$P_e = k_j \times Q\left(\sqrt{\frac{3P_i g_i}{2^j - 1}}\right), \text{ where } k_j = 4\left(1 - \frac{1}{2^{j/2}}\right) \tag{A1}$$

In Reference [13], it is proved that for $j \in [2, \infty), k_j \in [2, 4)$ the lower (for $k_j = 2$) and upper (for $k_j = 4$) bounds of P_e are tight. In Figure 7, we present the simulations comparison of Equation (A1) and its upper bound. The upper bound is rather close to the actual curve for all of the QAM modes. In the simulations, we were focused in the region from 10^{-3} to 10^{-9} error rates as it is the region of practical interest. Additionally, we present the case $j = 1$ which is not included in Reference [13].

In many papers, the upper bound of Equation (A1) for $k_j = 4$ is preferred to be used to describe the rate function for QAM modulation due to its simplicity (e.g. see Reference [3]). This is relation (6) rewritten here as Equation (A2).

$$P_e = 4Q\left(\sqrt{\frac{3P_i g_i}{2^j - 1}}\right) \tag{A2}$$

Solving Equation (A2) for P_i we get

$$P_i = \left[Q^{-1}\left(\frac{P_e}{4}\right)\right]^2 \left(\frac{2^j - 1}{3g_i}\right) \tag{A3}$$

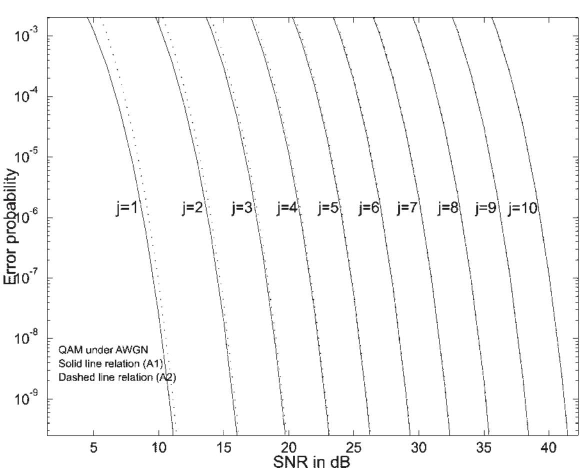

Fig. 7. Actual and upper bounded probability of error for several quadrature amplitude modulation (QAM) modes under AWGN.

The incremental energies then become

$$\Delta P_{i,j} = P_{i,j} - P_{i,j-1} = \frac{[Q^{-1}(P_e/4)]^2}{3g_i} 2^{j-1} \tag{A4}$$

From Equation (A4) can be easily extracted that $\left(\Delta P_{i,j+1}/\Delta P_{i,j}\right) = 2 > 1$ which is the proof of Proposition 1.

References

1. Haykin S. *Communication Systems*. John Wiley & Sons: New York, 2001.
2. Leke A, Cioffi J. A maximum rate loading algorithm for discrete multitone modulation systems. *IEEE Global Telecommunications Conference* 1997; **3**: 1514–1518.
3. Fischer RFH, Huber JB. A new loading algorithm for discrete multitone transmission. *IEEE Global Telecommunications Conference* 1996; **1**: 724–728.
4. Chow PS, Cioffi JM, Bingham JAC. A practical discrete multitone tranceiver loading algorithm for data transmission over spectrally shaped channels. *IEEE Transactions on Communication* 1995; **43**(2/3/4): 773–775.
5. Hughes-Hartogs D. Ensemble modem structure for imperfect transmission media. U.S. Patents *No. 4,679,227 (July 1987), 4,731,816 (Mar. 1988), and 4,833,706 (May 1989).*
6. Lai SK, Cheng RS, Letaief KB, Murch RD. Adaptive trellis coded MQAM and power optimization for OFDM transmission. *Vehicular Technology Conference* 1999; **1**: 290–294.
7. Campello J. Practical bit loading for DMT. *IEEE International Conference on Communications* 1999; **2**: 801–805.
8. Sonalkar RV, Shively RR. An efficient bit-loading algorithm for DMT applications. *IEEE Communications Letters* **4**(3): 80–82.
9. Baccarelli E, Biagi M. Optimal integer bit-loading for multicarrier ADSL systems subject to spectral-compatibility limits. *Signal Processing (EURASIP)* 2004; **84**(4): 729–741.
10. Gondran M, Minoux M. *Graphs and Algorithms.* John Wiley & Sons: New York, 1984.
11. Graham RL, Grotschel M, Lovasz L. *Handbook of Combinatorics*. MIT Press: Cambridge, 1995.
12. Edmonds J. Matroids and greedy algorithms. *Mathematical Programming* 1971; **1**: 127–136.
13. Kalet I. The multitone channel. *IEEE Transactions on Communications* 1989; **37**(2): 119–124.
14. Piazzo L. Fast algorithm for power and bit allocation in OFDM systems. *Electronics Letters* 1999; **35**(25): 2173–2174.
15. Proakis JG. *Digital Communication Systems*. McGraw-Hill: New York, 1989.
16. Czylwik A. Adaptive OFDM for wideband radio channels. *IEEE Global Telecommunications Conference* 1996; **1**: 713–718.
17. Krongold BS, Ramchandran K, Jones DL. Computationally efficient optimal power allocation algorithms for multicarrier communication systems. *IEEE Transactions on Communications* 2000; **48**(1): 23–27.
18. Bingham JAC. Multicarrier modulation for data transmission: an idea whose time has come. *IEEE Communications Magazine* 1990; **28**(5): 5–14.
19. Nemhauser GL, Wolsey LA. *Integer and Combinatorial Optimisation*. John Wiley & Sons: New York, 1988.
20. Goldsmith AJ, Chua SG. Adaptive coded modulation for fading channels. *IEEE Transactions on Communications* 1998; **46**(5): 595–602.
21. Keller T, Hanzo L. Adaptive orthogonal frequency division multiplexing schemes. *3rd ACTS Mobile Communication Summit* 1998; **2**: 794–799.

Authors' Biographies

Costas Assimakopoulos has received his diploma in Electrical and Computer Engineering from the Aristotle University of Thessaloniki, Greece, in 2000. He is currently a Ph.D. candidate in the same department. His research interests involve multicarrier transmission techniques and coding. He has also studied Power Lines used as communications media. He has participated COST 262 and INTERVUSE project. He was also scientific secretary of the 6th International Symposium on Power Line Communications and its Applications (ISPLC 2002) that was held on 27–29 March in Athens. He is a member of the IEEE and the Technical Chamber of Greece.

Fotini-Niovi Pavlidou received his Ph.D. in Electrical Engineering from the Aristotle University of Thessaloniki, Greece, in 1988 and the Diploma in Mechanical-Electrical Engineering in 1979 from the same institution. She is currently an associate professor at the Department of Electrical and Computer Engineering at the Aristotle University. Her research interests are in the field of mobile and personal communications, satellite communications, multiple access systems, routing and traffic flow in networks and QoS studies for multimedia applications over the internet. She is involved in many national and international projects in these areas (Tempus, COST, Telematics, IST) and she chaired the European COST262 Action on 'Spread Spectrum Systems and Techniques for Wireless and Wired Communications'. She has served as member of the TPC in many IEEE/IEE conferences and she has organized/chaired some conferences which include the 'IST Mobile Summit 2002', the 6th 'International Symposium on Power Lines Communications-ISPLC2002', the 'International Conference on Communications-ICT1998' etc. She is a permanent reviewer for many international journals. She has published about 80 papers in refereed journals and conferences. She is a senior member of IEEE, currently chairing the joint IEEE VT&AES Chapter in Greece.