

# A Comparative FDTD Study of Various PML Configurations for the Termination of Nonlinear Photonic Bandgap Waveguide Structures

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**Abstract** – A comparative investigation of high-performance PML absorbers for the termination of 2-D nonlinear photonic bandgap (PBG) waveguides, analyzed by the FDTD method, is conducted. Third-order nonlinear materials are considered, whereas existing effective permittivity schemes are properly implemented for the modeling of circular interfaces between linear and nonlinear media.

## INTRODUCTION

The numerical modeling of photonic bandgap materials has recently been a point of interest, due to their ability to control wave propagation at frequencies varying between microwave and optical [1]. However, structures mainly consisting of linear media have been analyzed so far.

In this paper, an attempt is made to extend the implementation of the FDTD method to photonic crystals comprising nonlinear materials. Specifically, utilizing a simple and accurate Z-transform based technique [2] for the simulation of the nonlinear media, various PML configurations (proper combination of nonlinear [3] and photonic crystal-based [4] PMLs) for the truncation of the waveguide are tested. The 2-D structure that is being studied consists of a periodic square lattice of air circles embedded in a nonlinear material (Fig. 1). The dielectric constants at the interfaces created between the air and the nonlinear material are computed via various effective permittivity schemes [5].

## FDTD FORMULATION OF THE NONLINEAR MATERIAL

The simulation of the nonlinear material is based on the following variation of the electric field constitutive equation

$$\epsilon_0 \epsilon_\infty \mathbf{E} = \mathbf{D} - \mathbf{P}_{NL}, \quad (1)$$

the nonlinear electric field polarization,  $\mathbf{P}_{NL}$ , being given by the convolution integral

$$\mathbf{P}_{NL} = \epsilon_0 \chi^{(3)} \mathbf{E}(t) \int_{-\infty}^{\infty} [a\delta(t-t') + (1-a)g_R(t-t')] \mathbf{E}^2(t') dt', \quad (2)$$

where delta function,  $\delta(t)$ , represents the Kerr effect and  $g_R(t) = (\tau_1^2 + \tau_2^2) / (\tau_1 \tau_2^2) \sin(t/\tau_1) \exp(-t/\tau_2)$  models the Raman scattering. Implementation of Z-transform in (2) converts the convolution to multiplication and allows the direct calculation of the nonlinear polarization. The final equations, thus, derived from (1) and (2) are

$$\mathbf{E}^n = \frac{\mathbf{D}^n + 2\epsilon_0 \chi^{(3)} a (\mathbf{E}^{n-1})^3}{\epsilon_0 \epsilon_\infty + \epsilon_0 \chi^{(3)} (1-a) \mathbf{S}_R^n + \epsilon_0 \chi^{(3)} a (\mathbf{E}^{n-1})^2}, \quad (3)$$

$\mathbf{S}_R^n = 2\exp(-T/\tau_2) \cos(T/\tau_1) \mathbf{S}_R^{n-1} - \exp(-2T/\tau_2) \mathbf{S}_R^{n-2} + T g_R(T) (\mathbf{E}^n)^2$  (4) which, along with Maxwell's curl equations in finite difference notation, constitute the solution algorithm.

## NONLINEAR PML ABSORBERS

The truncation of the waveguide is performed by means of, properly matched to the nonlinear material, PML absorbers,

the common basis of which is the standard (homogeneous) nonlinear PML. For the realization of the latter, conductivities related to the electric field displacement are introduced instead of the usual electric field conductivities. Moreover, the modified in stretched coordinates Maxwell's curl equations in frequency domain are the update equations inside the PML. The complex numbers  $s_x, s_y$  are assumed to be of the form

$$s_k(k) = [1 + (k/\delta)^n] [1 + \sigma_m (k/\delta)^n / j\omega], \quad (5)$$

where  $\sigma_m$  is constant,  $k = x, y$  and  $n=4$ .

The absorbing configurations compared in this paper are the homogeneous nonlinear PML and other inhomogeneous ones, in which the periodic pattern of the waveguide is retained inside the PML layers. Thus, not only the impedances but the wave numbers as well, are matched at the boundaries.

## RESULTS

The structure under test is a PBG waveguide formed by appropriately removing a row of circles from the initial periodic lattice (defect). It is excited by a TM modulated Gaussian pulse which propagates along the defect, as illustrated in Fig. 1. Fig. 2 depicts the relative error computed along a line transverse to the propagation direction, which lies inside the waveguide, 10 cells away from the PML. As expected, the preservation of the structure's periodic pattern inside the absorbing medium significantly increases its performance.

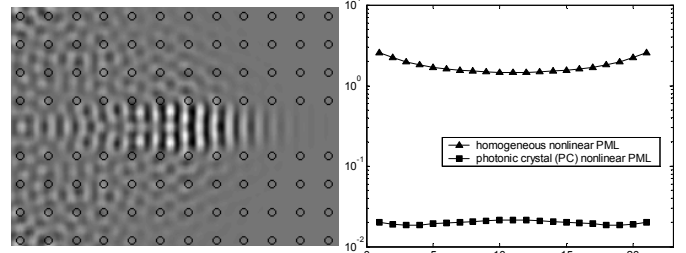


Fig. 1. Propagation along the defect.

Fig. 2. Local relative error (%).

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